One Bit to Rule Them All

Binarizing the Reconstruction in 1-bit Compressive Sensing

Thomas Feuillen, Mike Davies, Luc Vandendorpe, Laurent Jacques

December 3rd, 2020

STRUCTURE

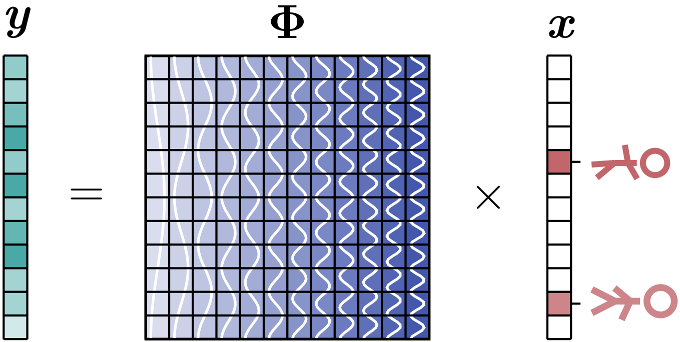
Quantized Measurements

- Linear model
- Quantizer
- Reconstruction algorithm

Quantized Reconstruction

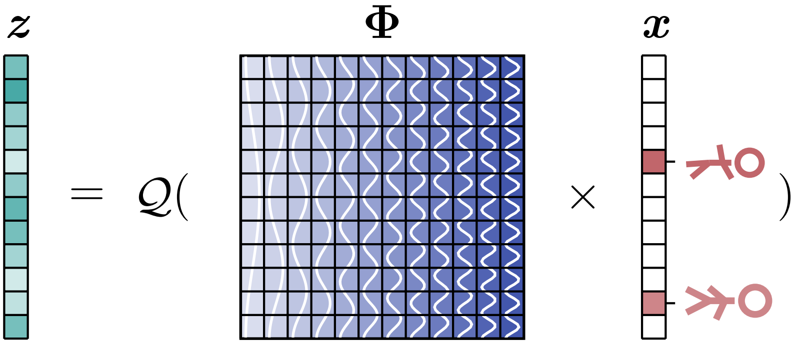
- Idea
- Recovery guarantee
- Factorized model

QUANTIZED MEASUREMENTS



We consider :

- $\boldsymbol{y} = \boldsymbol{\Phi}\boldsymbol{x}$
- $\boldsymbol{\Phi} \in \mathbb{C}^{M \times N}$
- K targets
- $\boldsymbol{x} \in \mathbb{C}^N$, $\|\boldsymbol{x}\|_0 := |\operatorname{supp} \boldsymbol{x}| \leq K \ll N$



We consider :

- $\boldsymbol{z} = \mathcal{Q}(\boldsymbol{\Phi}\boldsymbol{x})$
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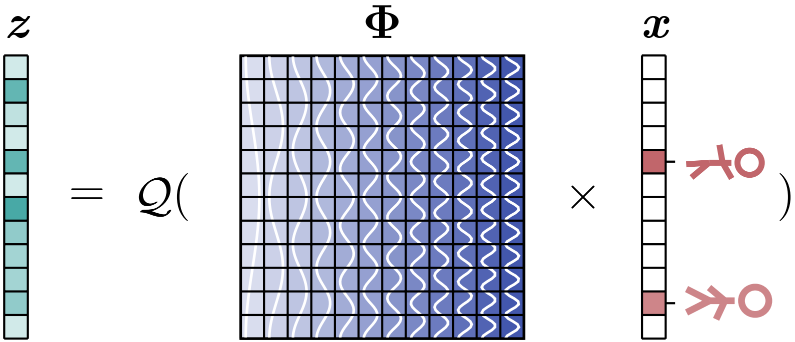

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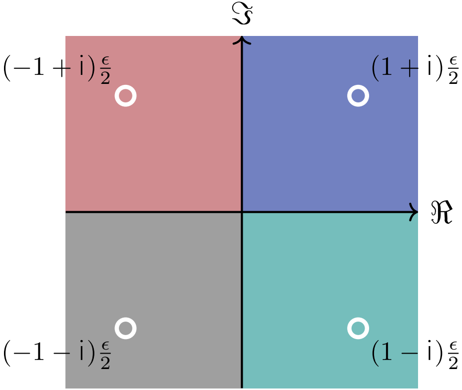
Which $\mathcal{Q}(\cdot)$ should we use ?

Complex 1-bit quantizer:

$$\mathcal{Q}_\epsilon(\lambda) = \frac{\epsilon}{2}\,\mathrm{sign}\left(\Re(\lambda)\right) + \mathrm{i}\,\frac{\epsilon}{2}\,\mathrm{sign}\left(\Im(\lambda)\right),$$

with $\epsilon > |\boldsymbol{\lambda}|_\infty$



Ambiguity ?

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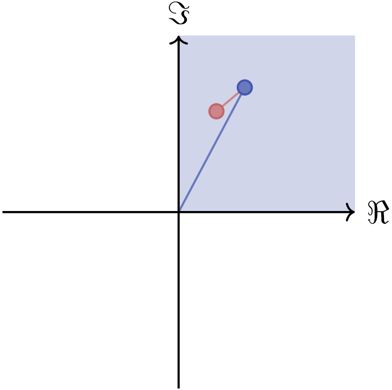
When $\mathcal{Q}_\epsilon(\boldsymbol{\Phi}\boldsymbol{x}_0) = \mathcal{Q}_\epsilon(\boldsymbol{\Phi}\boldsymbol{x}_1)$

- For $K > 1$, with $\operatorname{supp}(\boldsymbol{x}_0) \neq \operatorname{supp}(\boldsymbol{x}_1)$

$$r_0[m] = 1\, e^{-\mathrm{i}\psi_{n_0}}\, e^{-\mathrm{i}2\pi f_m \frac{2R_0}{c}}$$

$$r_1[m] = 1\, e^{-\mathrm{i}\psi_{n_0}}\, e^{-\mathrm{i}2\pi f_m \frac{2R_0}{c}} + \gamma\, e^{-\mathrm{i}\psi_{n_1}}\, e^{-\mathrm{i}2\pi f_m \frac{2R_1}{c}}$$

[Feuillen et al, 1811.11575]



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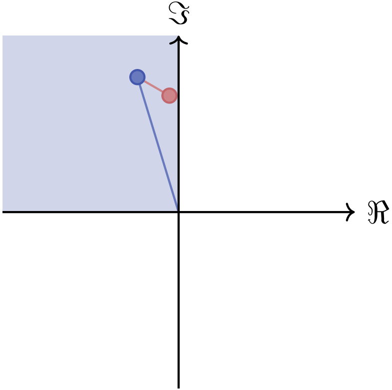

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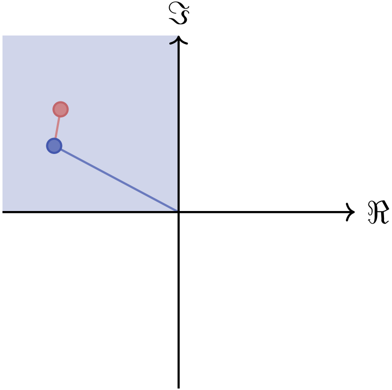

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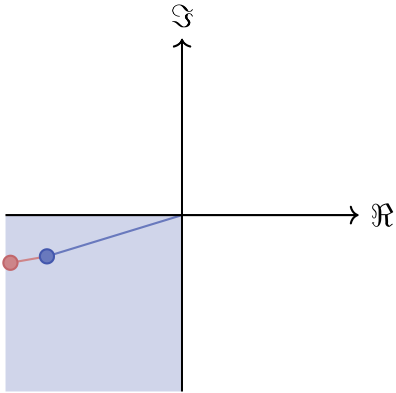

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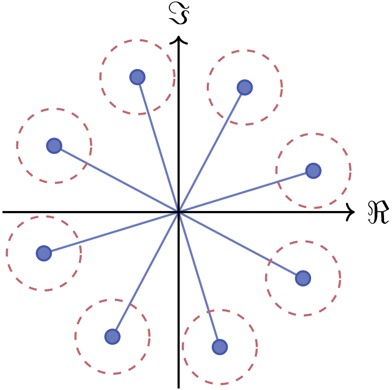
$\Rightarrow$ **Bound on the reconstruction**



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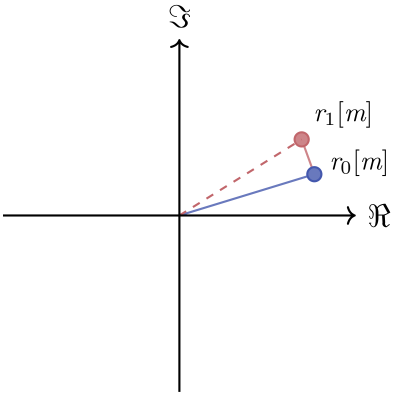

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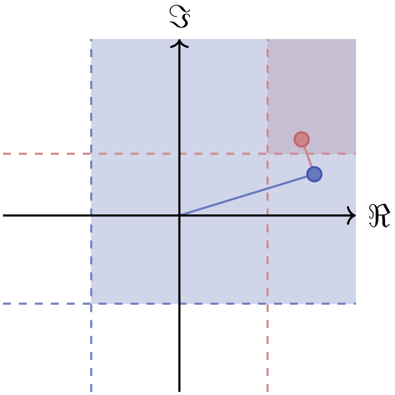

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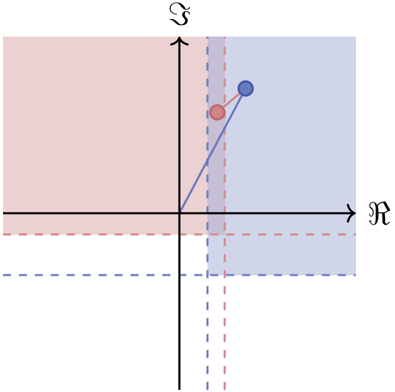

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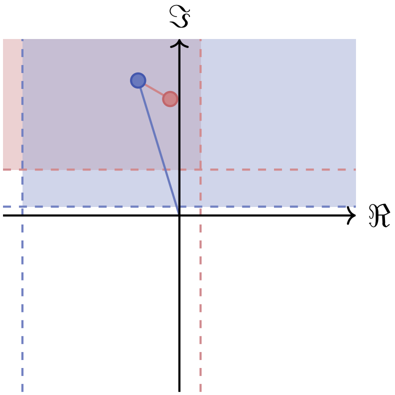

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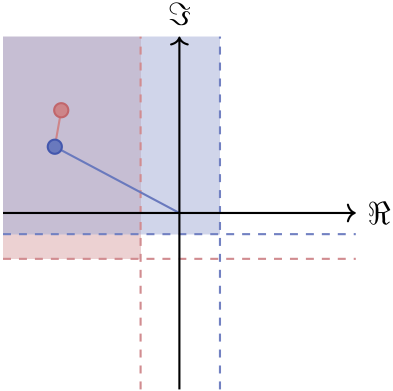

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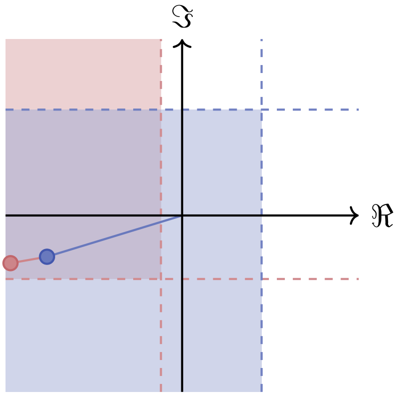

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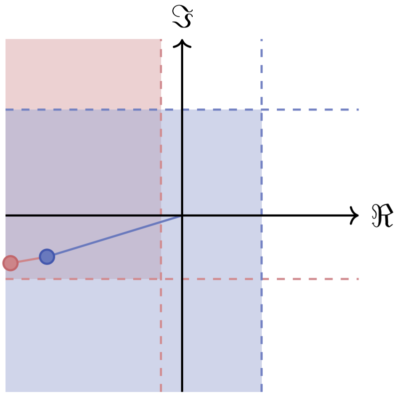

PROJECTED BACK PROJECTION

The PBP estimate :

$$\hat{\boldsymbol{x}} = \frac{1}{m}\mathcal{H}_K(\boldsymbol{\Phi}^H \boldsymbol{z})$$

- $\mathcal{H}_K(\cdot)$ is the Hard Thresholding operator

- Pros:
  - Simple
  - No explicit use of dithering
    - Ease of implementation in hardware
  - Reconstruction error : $\|\hat{\boldsymbol{x}} - \boldsymbol{x}\| = O\big((1+\delta)\sqrt{K/m}\big)$ [Xu and Jacques, 1801.05870]

- Cons:
  - No explicit use of dithering
    - Low power targets are drowned in the dither
  - Poor reconstruction when K grows

QUANTIZED RECONSTRUCTION

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The processing is still done in **High** resolution

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Can we also lower the resolution of the Back-Projection ?

Advantages

- Possibly more efficient processing
- 1-bit computation can be simplified
- Applications: low cost, multi sensors, ...

Challenges

- Effect of quantization ?
- Impact on performances ?
- Reconstruction algorithm ?
- Reconstruction guarantees ?

The backprojection becomes

$$\hat{\boldsymbol{x}} = \frac{1}{m}\mathcal{H}_K(\mathcal{Q}_\nu(\boldsymbol{\Phi}^H + \boldsymbol{\xi})\mathcal{Q}_\epsilon(\boldsymbol{\Phi}\boldsymbol{x} + \boldsymbol{\xi}))$$

- 1-Bit : $\nu \geq 2\|\boldsymbol{\Phi}^H\|_\infty$
- With dithering

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ONE BIT TO RULE THEM ALL

BINARIZING THE RECONSTRUCTION IN 1-BIT COMPRESSIVE SENSING

WHAT CAN WE DO ?

For $\boldsymbol{\Psi}^H := \mathcal{Q}_\nu(\boldsymbol{\Phi}^H + \boldsymbol{\xi})$, and $\mathcal{S}$ the estimated support

$$\|\boldsymbol{x} - \hat{\boldsymbol{x}}\|_2 = \|\boldsymbol{x} - \frac{1}{m}(\boldsymbol{\Psi}^H \mathcal{Q}_\epsilon(\boldsymbol{\Phi}\boldsymbol{x} + \boldsymbol{\xi}))_{\mathcal{S}}\|_2$$

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$$
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&\leq \|\boldsymbol{x} - \frac{1}{m}(\boldsymbol{\Phi}^H \mathcal{Q}_\epsilon(\boldsymbol{\Phi}\boldsymbol{x} + \boldsymbol{\xi}))_{\mathcal{S}}\|_2 + \frac{1}{m}\|((\boldsymbol{\Phi}^H - \boldsymbol{\Psi}^H)\mathcal{Q}_\epsilon(\boldsymbol{\Phi}\boldsymbol{x} + \boldsymbol{\xi}))_{\mathcal{S}}\|_2
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We need to **upper-bound** this term

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- Leverage the result in **expectation**

$$\mathbb{E}\{\boldsymbol{\Psi}^H\} = \boldsymbol{\Phi}^H$$

  - Use **Concentration of Measure** (like Hoeffding)

UNIFORM RESULT

After a bit of work we obtain :

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Theorem: For all K-sparse unit vector $\boldsymbol{x} \in \mathbb{C}^N$ and for a measurement matrix $\boldsymbol{\Phi} \in \mathbb{C}^{m\times N}$ that follows the (ℓ_2, ℓ_2)-RIP$(2K, \delta)$ and given the dithered 1-bit quantizer, and for all support $\mathcal{S}$ of size K, one can bound

$$\frac{1}{m}\|((\boldsymbol{\Phi}^H - \boldsymbol{\Psi}^H)\mathcal{Q}_\epsilon(\boldsymbol{\Phi x} + \boldsymbol{\xi}))_{\mathcal{S}}\|_2 \leq C\sqrt{K}\gamma(1 + \frac{1}{\nu\epsilon})$$

with a probability exceeding $1 - C\exp\left(-c\frac{\gamma^2 m}{\nu^2\epsilon^2}\right)$.
Provided

$$m \geq c\gamma^{-2}K\log(\frac{N\sqrt{m}\nu^2\epsilon}{\gamma})$$

Which finally gives :

$$\|\boldsymbol{x} - \frac{1}{m}(\mathcal{Q}_{\nu}(\boldsymbol{\Phi}^{H} + \boldsymbol{\xi})\mathcal{Q}_{\epsilon}(\boldsymbol{\Phi}\boldsymbol{x} + \boldsymbol{\xi}))_{\mathcal{S}}\|_2 \leq \mathcal{O}(\sqrt{\frac{K}{m}})$$

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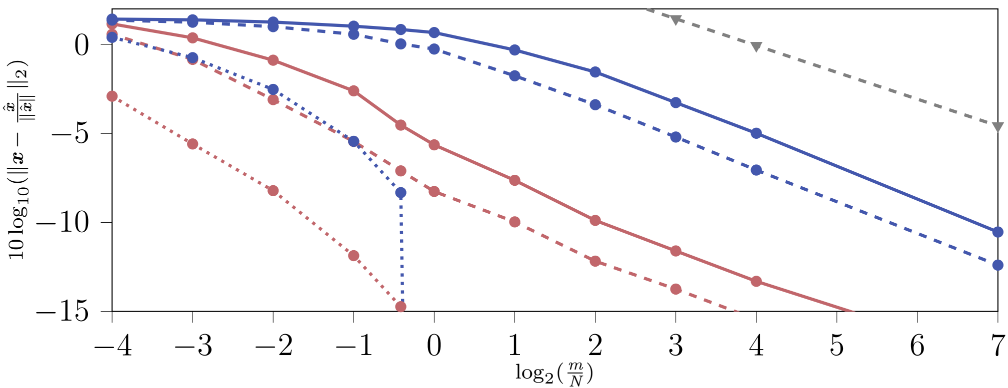
QPBPQ has the same convergence rate as PBPQ

Impact of the 1-Bit dithered BP for Fourier



- K=2, K=10
- $N = 256$
- Dotted = PBP, Dashed = PBPQ
- Solid = QPBPQ

CAN WE EVEN GO FURTHER ?

QUANTIZED FACTORIZED BACKPROJECTION

$$\mathcal{Q}_\nu(\mathbf{\Phi}^H + \boldsymbol{\xi})\boldsymbol{z}$$

- Simple Matrix Multiplication
- $\mathcal{O}(mN)$ complexity

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- $\mathcal{O}(m \log_2(N))$ complexity (FFT)

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Can we leverage both the low Resolution and low Complexity ?

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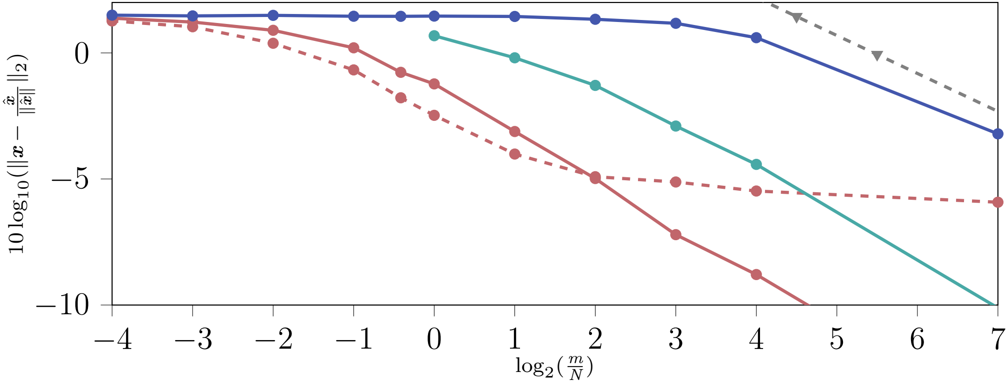
YES, Paper (hopefully) coming soon ;P

For different Φ



- QFourier, QFFT, QGaussian
- $N = 256$
- Solid = Dithered
- Dashed = Non-Dithered

CONCLUSION

CONCLUSIONS

- Uniform recovery for any RIP matrix
- Low losses in performances
- Can be extended to classic measurements
- Dithering is necessary
- Can leverage low complexity of factorizable model (QFFT)

Any questions ?

thomas.feuillen@uclouvain.be
thomas-feuillen.com
@Tfeuillen

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