An extreme bit-rate reduction scheme for 2D radar localization

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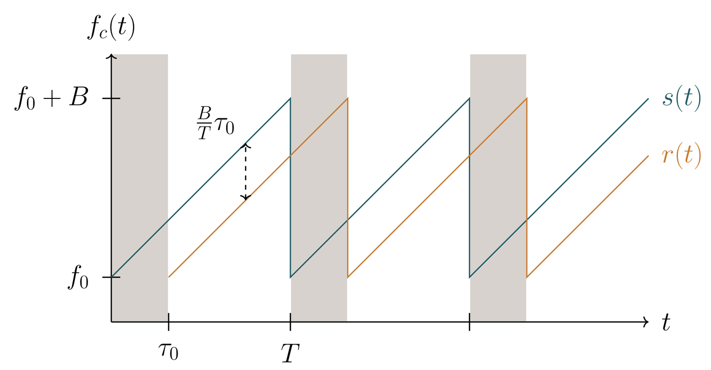
Objectives

- **FMCW radar** are used for **2D localization** for security and road monitoring applications.
- **Radar signal processing** requires **high** sampling frequency:
  - High **costs** for high resolution **ADC** with high sampling rate.
  - High **bit-rate** for multiple antennas.
- **1-bit Compressive Sensing** enables:
  - **Cost effective** implementation.
  - **Lower bit-rate**.

Radar Emission/Reception Model

- **Frequency Modulated Continuous Wave Radar**



- **Transmitted signal** for one transmitting antenna:

$$s(t) = \sqrt{P_t}\,\exp\left(2\pi \mathrm{i}\left(\int_0^t f_c(\xi)\mathrm{d}\xi + \phi_0\right)\right),$$

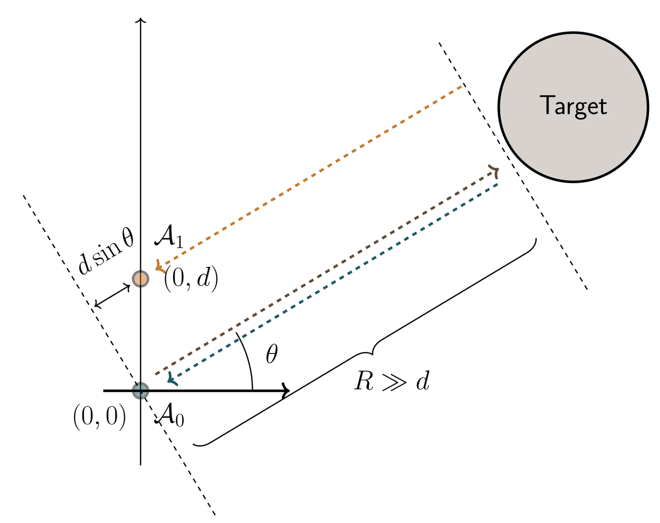
  - $f(t)$ is the chirp carrier function :

$$f_c(t) = f_0 + B\left(\frac{t}{T} \bmod 1\right),$$

  where T is the chirp rate and B the bandwidth.
- For a static target at range R and angle θ, the received signals on $\mathcal{A}_p$ ($p \in \{0,1\}$) is

$$\gamma_p(t) = A\,s(t - \tau_p),$$

  - A is the complex received amplitude
  - Far-field approximation: $\tau_p = c^{-1}(2R + p\,d\sin\theta)$ is the round-trip time between the transmitting antenna in $(0,0)$, the target and $\mathcal{A}_p$, c is the speed of light.



Inverse problem formulation

- **Assumption**: purely additive model of the signals reflected by the targets
- **Multi-target**: K targets
- **Received signal** after sampling ($\frac{B}{T}t \to f_m$) becomes:

$$\Gamma_{mp} = x\,e^{-\mathrm{i}2\pi f_m \tau_p} \approx x\,e^{-\mathrm{i}2\pi f_m \frac{2R}{c}} e^{-\mathrm{i}2\pi f_0 \frac{pd\sin\theta}{c}}$$

  - f_m are chosen at **semi-random**, *i.e.*, for M measurements, $\lfloor\frac{M}{N}\rfloor$ complete ramps and $M \bmod N$ frequencies are uniformly sampled at random on the last ramp
- Recasting in a linear system :

$$\boldsymbol{\Gamma} = [\boldsymbol{\gamma}_1, \boldsymbol{\gamma}_2] = \boldsymbol{\Phi}\,[\boldsymbol{x}, \boldsymbol{G}\boldsymbol{x}],$$

  - $\boldsymbol{x} \in \mathbb{C}^N$ encodes the range profile
  - $\|\boldsymbol{x}\|_0 := |\mathrm{supp}\,\boldsymbol{x}| \le K \ll N$
  - $\boldsymbol{\Phi} = \{e^{-\mathrm{i}\frac{4\pi}{c}f_m R_n}\}_{mn} \in \mathbb{C}^{M\times N}$ is the *range* measurement matrix
  - $\boldsymbol{G} = \mathrm{diag}(e^{\mathrm{i}\frac{2\pi}{c}f_0 d\sin\theta_1}, \cdots, e^{\mathrm{i}\frac{2\pi}{c}f_0 d\sin\theta_N})$ with $\mathrm{supp}\,(\boldsymbol{\theta}) = \mathrm{supp}\,(\boldsymbol{x})$

$\Rightarrow$ $\boldsymbol{G}$ is the phase difference between the first and second receiving antennas.

Channel dropping model

- After coherent demodulation :
  - Complex signals are represented using 2 (IQ) channels.
  - Requires two ADCs per antenna.
- To reduce bit-rate, half of the channels are dropped :

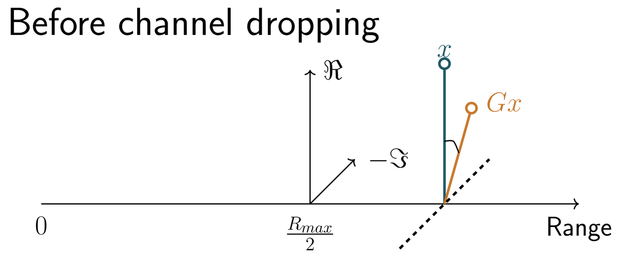
$$\boldsymbol{y}_1 = \mathrm{Re}\{\boldsymbol{\gamma}_1\}, \quad \boldsymbol{y}_2 = \mathrm{iIm}\{\boldsymbol{\gamma}_2\}$$

- Given the signal model, we exploit :

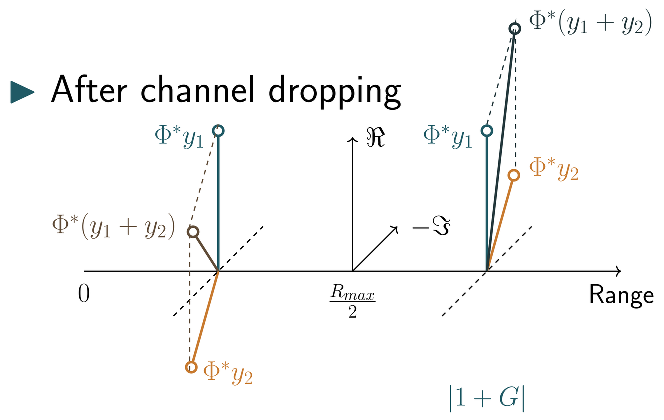
$$\begin{aligned}\hat{\boldsymbol{y}} &= \boldsymbol{y}_1 + \boldsymbol{y}_2 \\ &= (1+G)\,e^{-\mathrm{i}2\pi f_m \frac{2R}{c}} \\ &\quad + (1-G^*)\,e^{\mathrm{i}2\pi f_m \frac{2R}{c}}\end{aligned}$$

- So, if $\boldsymbol{\theta}$ small, $\mathrm{supp}\,(\Phi^*\hat{y}) \approx \mathrm{supp}\,(x)$ with some ambiguities when $K > 1$.

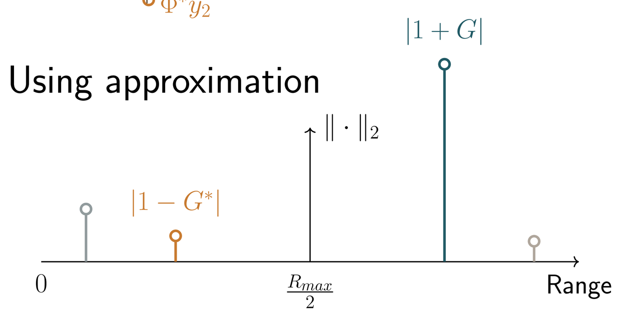
- Before channel dropping



- After channel dropping



- Using approximation

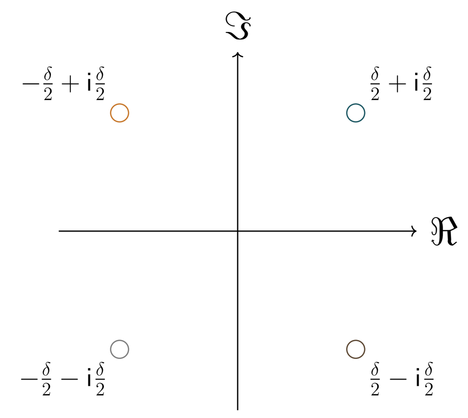


Quantization

- Uniform scalar quantizer:

$$\begin{aligned}\lambda \in \mathbb{R} &\mapsto \mathcal{Q}(\lambda) = \delta\left\lfloor\frac{\lambda}{\delta}\right\rfloor + \frac{\delta}{2} \\ \lambda \in \mathbb{C} &\mapsto \mathcal{Q}(\lambda) = \mathcal{Q}(\Re(\lambda)) + \mathrm{i}\mathcal{Q}(\Im(\lambda))\end{aligned}$$

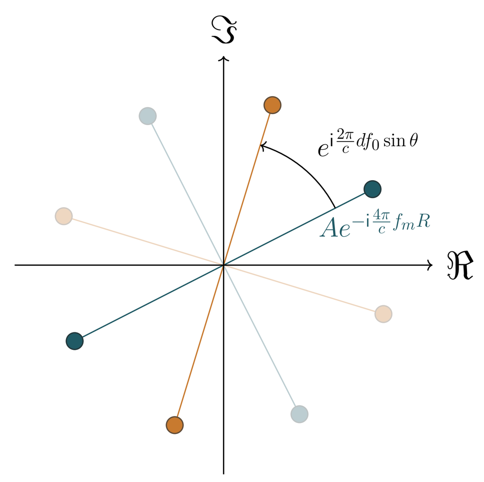
  - For $\delta > |\boldsymbol{\lambda}|_\infty \to \mathcal{Q}(\cdot)$ is 1-bit



Indistinguishability

- Ambiguities when $\mathcal{Q}(\boldsymbol{\Phi x}) = \mathcal{Q}(\boldsymbol{\Phi G x})$
  - For $\boldsymbol{\theta}$ small $\to \boldsymbol{G} \approx \mathrm{Id}$
  - For $\boldsymbol{\theta}$ big $\to$ certain R_n induce ambiguities

  $\Rightarrow$ Bound on the reconstruction
    - Part of the information is removed
    - Even for large M



Dithering

- The distance preservation properties of the *RIP* can be inherited by using a dither.

$$\mathsf{A}(\cdot) = \mathcal{Q}(\boldsymbol{\Phi}\cdot + \boldsymbol{\xi})$$

  where $\boldsymbol{\xi} \in \mathbb{C}^m$ defined as $\xi_i = \xi_i^{\mathbb{R}} + j\xi_i^{\mathbb{I}}$, with $\xi_i^{\mathbb{R}}, \xi_i^{\mathbb{I}} \sim \mathcal{U}(-\frac{\delta}{2}, \frac{\delta}{2})$.
- Idea behind it :

$$\mathbb{E}\big[\mathsf{A}(\boldsymbol{x})\big] = \boldsymbol{\Phi x}$$

- The Quantized model with channel dropping :

$$\boldsymbol{Z} = \mathbf{A}^D(\boldsymbol{X}) := \big(\mathrm{Re}\{\mathbf{A}(\boldsymbol{x}_1)\}, \mathrm{iIm}\{\mathbf{A}(\boldsymbol{x}_2)\}\big).$$

Projected Back Projection

The PBP estimate :

$$\hat{\boldsymbol{X}} = \frac{1}{M}\boldsymbol{\Phi}^*\boldsymbol{Z}$$

The support is estimated using the approximation :

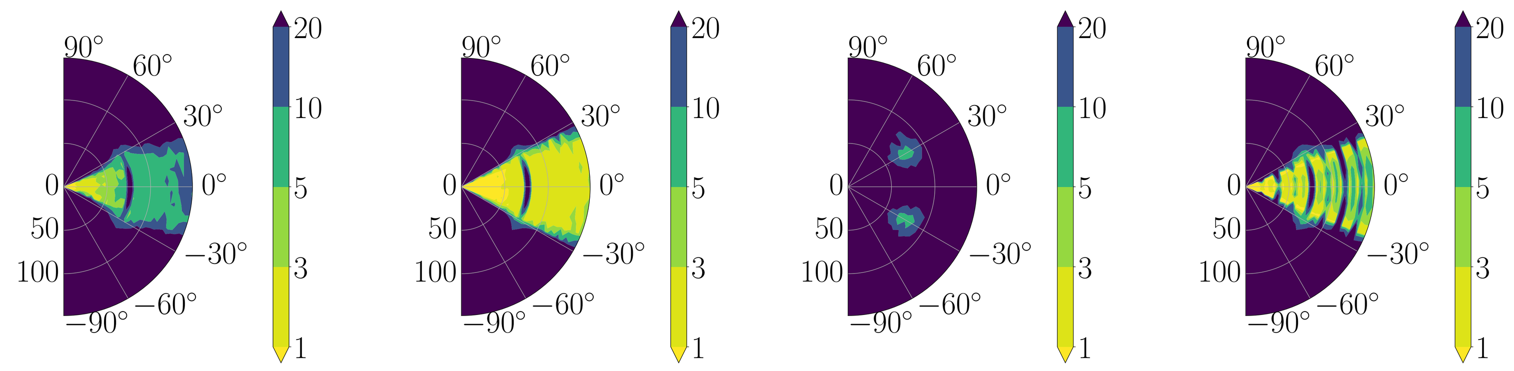
$$\hat{T} = \mathrm{supp}\,\left(\mathcal{H}_K^{\mathsf{Sym}}(\hat{\boldsymbol{x}}_1 + \hat{\boldsymbol{x}}_2)\right)$$

- $\mathcal{H}_K^{\mathsf{sym}}(\cdot)$, is the hard thresholding that projects to the common support domain, removing the ambiguity coming from the approximation.
- Angle Estimation:
  - Using $\hat{\boldsymbol{X}}$ and $\hat{T}$, $\theta_n = \arcsin\left(\frac{c}{2\pi f_0 d}\angle(\hat{x}_1[n]^*\hat{x}_0[n])\right)$.
- Fitted for the considered applications :
  - Simple
  - No explicit use of dithering

Results

- **Monte Carlo** simulations are performed for different scenarios (R,θ) with no noise.
  - $\min_k |Re^{\mathrm{i}\theta} - \hat{R}_k e^{\mathrm{i}\hat{\theta}_k}|_2$
  - $N = 256$
  - Bit-rate : $\mathcal{B} = M \times \#Channels \times b$
  - Signal amplitude :
    - Strong target : 1
    - Weak target : $\mathcal{U}(0,1)$
- **One target : 1-bit**

Dithering | Dithering | Dithering Classic PBP | No Dithering



- $M/N = 20\%$
- $M/N = 200\%$
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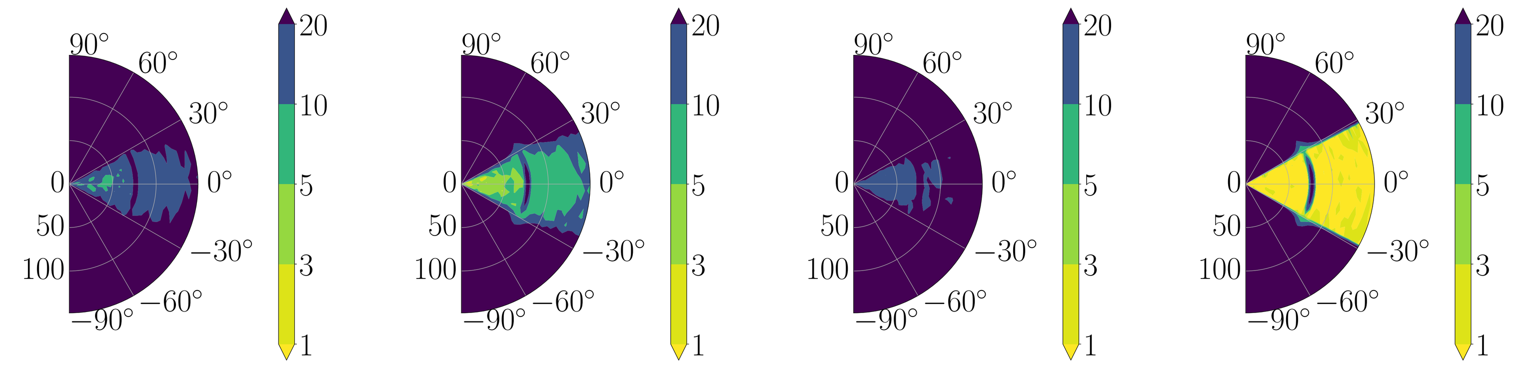
- $M/N = 200\%$

- **Two targets : 1-bit vs No Quantization**

No Dithering | Dithering | No quantization | No quantization



- $M/N = 200\%$, $\mathcal{B} = 3.1\%$
- $M/N = 200\%$, $\mathcal{B} = 3.1\%$
- $M/N = 6.25\%$, $\mathcal{B} = 3.1\%$
- $M/N = 100\%$, $\mathcal{B} = 50\%$

Conclusions

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- 2D localization is achievable
- 1-bit quantization
- 1 channel per antenna
- Dithering solves artifacts
- Simulations consistent with theory

Future works

- Effect of the noise
- More advanced algorithms
- Larger array
- Actual implementation
- Imaging

References

- C. Xu and L. Jacques, *Quantized compressive sensing with RIP matrices*, 2018
- T. Feuillen, C. Xu, L. Vandendorpe, L. Jacques, *1-bit Localization Scheme for Radar using Dithered Quantized Compressed Sensing*, 2018.